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TECHNICAL REPORT  
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# **MULTIECHELON NETWORK MODEL AND HEURISTIC FOR THE COMBAT SERVICE SUPPORT SUPPLY SYSTEM (CS4)**

by

**Steven J. Yuhaski, Jr.**

August 1994

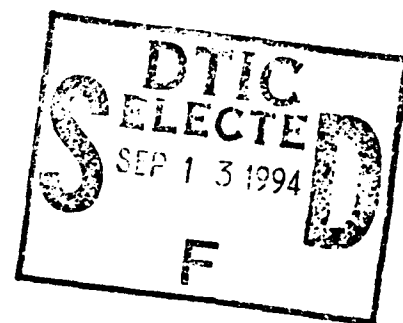
Final Report

December 1988 - August 1991

**94-29638**



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1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE August 1994	3. REPORT TYPE AND DATES COVERED Final Dec 1988 - Aug 1991	
4. TITLE AND SUBTITLE MULTIECHELON NETWORK MODEL AND HEURISTIC FOR THE COMBAT SERVICE SUPPORT SUPPLY SYSTEM (CS4)			5. FUNDING NUMBERS PE: 61102 WU: E00 PR: AH52 Task: 04	
6. AUTHOR(S) Steven J. Yuhaski, Jr.				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army Natick RD&E Center Attn: SATNC-AA Kansas St. Natick, MA 01760-5015			8. PERFORMING ORGANIZATION REPORT NUMBER NATICK/TR-94/030	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSORING / MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES Approved for public release, distribution unlimited				
12a. DISTRIBUTION / AVAILABILITY STATEMENT			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  The Quartermaster School through the Joint Technical Staff (JTS) established the need, through brainstorming, for the development of a multiechelon inventory model of a combat service support supply system (CS4). The participation of the U.S. Army Natick RD&E Center's Sustainability Directorate was instrumental in establishing this need. The model will have the potential to enhance the logistical capabilities in any given theater of operations. Since the possibility of an intense and prolonged military conflict is always present in any theater of operations, supply/distribution logistics systems must be able to respond effectively to enable the troops to initiate and sustain the required military operations. Preparedness and sustainability can be enhanced by lowering the overall inventory levels through more efficient transportation scheduling; this would increase the overall rate of rotation of perishable rations in inventories within the theater. The method of dynamic flows and a minimum-cost flow network algorithm were employed to model the logistics system. A heuristic was developed to determine feasible and efficient solutions to the multicommodity flows of the network model. Since the application of the model is multiobjective, it can also be useful in reducing pre-positioning inventory levels in addition to reducing the burden on transportation systems within a given theater.				
14. SUBJECT TERMS ROTATION RATES HEURISTIC ALGORITHMS LOGISTICS SYSTEMS		MULTIECHELON INVENTORY INVENTORY MODELS COMBAT SERVICE SUPPORT SUPPLY SYSTEM SUSTAINABILITY PREPAREDNESS INVENTORY REDUCTION		15. NUMBER OF PAGES 36
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED		18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED		16. PRICE CODE
		19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED		20. LIMITATION OF ABSTRACT

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## PREFACE

The work described in this report was authorized under Project No. 1L161102AH5204E00, Expert Multiechelon CS4. The work was performed from December 1988 to August 1991.

The author wishes to thank Dr. Paul Leitch for his part in initiating the project and his informed guidance, Dr. Michael Ketcham and David Airth of the Dept. of Industrial Engineering and Operations Research at the University of Massachusetts - Amherst for their contributions in useful background information and perspective, Dale Malabarba for his insightful comments about the contents and structure of the report, William Chevalier and Peter DeCosta for their thorough and insightful peer reviews, and Marcia Lightbody for her guidance and her enlightening editorial comments.

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# MULTIECHELON NETWORK MODEL AND HEURISTIC FOR THE COMBAT SERVICE SUPPORT SUPPLY SYSTEM (CS4)

## INTRODUCTION

### Background

A series of brainstorming sessions was held by the Quartermaster School (QMS) through the Joint Technical Staff (JTS) at Natick Research, Development and Engineering Center. The involvement of the Food Engineering Directorate (FED) in the brainstorming sessions led to serious speculation that the development of a multiechelon inventory model of a combat service support supply system (CS4) may have the potential to enhance the logistical capabilities in any given theater of operations.

In any theater of operations, the possibility of a military conflict is always present. Not only must the supply/distribution system of a theater be able to react effectively to the needs of the troops, it must also have the capability to sustain an adequate flow of supplies to the field troops, in a coordinated manner, in the event of a prolonged and intense conflict. Since the information obtained by intelligence gathering is not always accurate, complete, or current enough under constantly changing circumstances, an adequate degree of preparedness and sustainability is essential for the security of a theater. Without adequate logistical planning and support, a sudden and intense conflict can prevent needed subsistence from reaching field troops. Particularly critical is the need for field troop supply support at the lower echelons from the upper echelons of the theater. Not only must the supply/distribution system be able to react effectively to the needs of the troops, it must also have the capability to sustain an adequate flow of supplies to the field troops, in a coordinated manner, in the event of a prolonged and intense conflict.

Supply/distribution policies that are currently in effect in theaters tend to be static; that is, they do not account for the stochastic and dynamic demands that can adversely influence a supply/distribution system in times of military conflict. Thus, under the current policies, the likelihood of problems in the logistics system that are severe enough to compromise the military effectiveness of a theater is significant. Logistics problems that are of paramount concern relate to the possibility of stockouts occurring at any echelon in the theater as the result of sudden surges in demand; these are especially critical at the lowest echelons where the need for supplies is typically most highly concentrated.

There are many analytical methods available to cope with logistics problems that are related to stochastic and dynamic behaviors, such as demand. Operations research has many basic methodologies; however, the method or combination of methods applied is often influenced by judgment, perspective, and goals of the analyst. Possibilities that exist within a broad range of techniques include inventory analysis, mathematical programming, just-in-time inventory methods, and network analysis.

## Objective

The objective of this effort is to identify the important aspects of logistical supply/distribution problems with respect to military theaters of operation and develop an analytical approach that can be used to ensure that troop demands for perishable rations are met. The goal is to reduce inventory levels so that the rations are rotated more quickly and to reduce the use of transportation resources, i.e., vehicles, fuel, roadways, railways, etc.

## Approach

A network analysis approach is used to cut inventory levels and transportation activities throughout the theater. The method of choice is a dynamic flows expansion of a minimum-cost flow method. The out-of-kilter (OKA) algorithm is used primarily because lower bounds for the arc (route) flows not only ensure geographical coordination in the logistics network but also enforce temporal coordination as well. Solution methods to exogenous source limitations and pre-positioning can also be accounted for by use of the method of dynamic flows.

Since a major aspect of the modelling is the existence of different types of rations, the flows are multicommodity. A heuristic is developed to deal with the multicommodity flows version of the problem.



## THEATER RATION DISTRIBUTION SYSTEM

This section of the report outlines the aspects of the ration supply/distribution system that lead to the development of the logistics model. The important aspects of the system are its operating policies, its structure, and its problematic issues. The application of the model is also described. Information about the ration distribution systems in military theaters was drawn from numerous discussions with experienced personnel and from field manuals.<sup>1-5</sup>

### Operating Policies

Current ration distribution policies include a "push" and a "pull" component. The push component projects future demands and moves supplies ahead in an effort to satisfy the projected demands. The push component moves supplies (from both CONUS and OCONUS) to facility distribution locations. Currently the supplies are moved by long-range transportation to any given theater; this is usually done by sea in 30-day palletized lots. The lots are then moved by theater transportation systems, which are usually truck and/or rail, to the distribution facilities for future requirements. The pull component of the distribution process reacts to immediate demands that occur at the unit level, that is, primarily at or near the front line of troops (FLOT). Within the context of this project, the term "unit" refers generically to military personnel at the brigade level or lower (see Figure 1). Consequently, the (ration) supplies are "pulled" to the demand locations by requisitions. During this process the units review the supply requirements on a daily basis, if possible; requisitions are prepared by the units at the demand locations; the supplies are transported (i.e., "pulled") usually by truck or rail from the storage facilities to the demand locations.

In addition to the push and pull activities, supplies are held in inventory to ensure preparedness. Preparedness requirements determinations are based on troop strengths for a given level and duration of conflict and current feeding policies (such as MRE to T-Ration ratios, etc.).

### Ration Distribution Structure

The structure that is associated with rations distribution in a theater is shown in Figure 1. The thin arrows represent information flow and the thick arrows represent the flow of (Class 1) ration supplies. Typically the information flow starts at the field unit and reaches TA MMC (Theater Area Materiel Management Center), which is the integrated inventory management center for the entire theater. COSCOM MMC (Corps Support Command Materiel Management Center) is the functional control center for supply and maintenance management; it relies heavily on the computer capabilities of COSCOM (Corps Support Command). COSCOM MMC directs storage and distribution of subsistence, reviews and analyzes demands, and balances the workloads of subsistence supply units at or near the FLOT. DMMC (Division Materiel Management Center) provides centralized and integrated materiel management support for Class I supplies. DMMC is assigned to DISCOM (Division Support Command). The MMC that exists in each of the military echelons, i.e., the theater, corps, and division

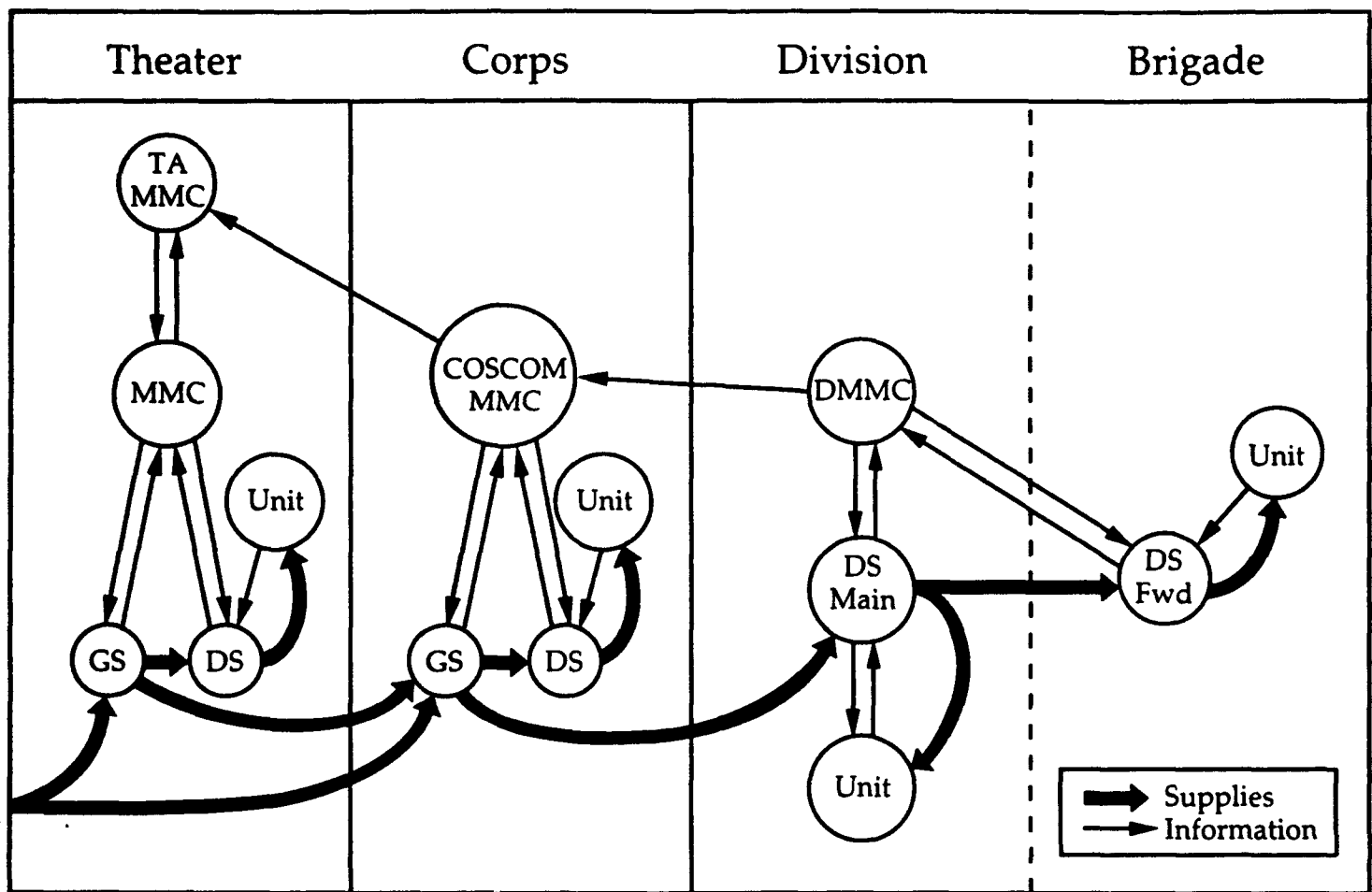


Figure 1.  
Flow of Class 1 Supplies in a Theater

TA MMC = Theater Area Materiel Management Center

MMC = Materiel Management Center

GS = General Support

DS = Direct Support

COSCOM MMC Corps Support Command Materiel Management Center

DMMC = Division Materiel Management Center

DS Main = Direct Support Main

DS Fwd = Direct Support Forward

levels, provides its corresponding level of coordination of supply activities. The lower (military) echelons provide increasingly more detailed administration of supplies in their own areas. The MMC of each area communicates with both General Support (GS) and Direct Support (DS) units and provides logistical support for these units. The supplies are physically handled only by the GS and DS units (under normal circumstances). In Figure 1, note that DS Main provides direct support to the divisional units and sends supplies to DS Fwd (DS Forward) to be distributed to all units at and below the brigade echelon. Also note that supply routes that are exogenous to the theater typically lead to the GS units at the theater and corps level; these supply sources can originate from an outside nation or from host nation support.

The generic structure of a distribution node is shown in Figure 2. As in the previous figure, information flow is represented by a thin arrow and (ration) supply flow is indicated by a thick arrow. Supplies from various sources that flow into the area are inventoried and distributed by the GS and DS units. The field units in the area are supplied by the DS units and report supply usage and requisition back to the DS unit. The GS can supply the DS units in the area and the GS units in lower echelons. The generic structure of each location is represented by a node in the logistics network.

#### Problematic Issues

Since the possibility of an intense military conflict always exists at or near military theaters, the ration supply/distribution system must be able to react appropriately to provide subsistence to troops. Therefore, the objective of the logistics network model is to help provide quantitative decision making capabilities to upper-echelon MMC users in the theater. The top priority is to meet the demands for ration distribution — especially the demands of the field units in the lower echelons

There are two major types of criteria that define the problematic issue of logistical support: preparedness and sustainability. Preparedness, in the logistical context, is the ability of the logistics system to react effectively and decisively to any adverse circumstance that may occur with little or no advance warning. Preparedness includes the ability of personnel to make decisions about the location and amount of pre-positioned supplies, and the ability to have transportation and distribution resources in place and in order. The goal of preparedness is to provide logistical support until sustainability capabilities are achieved. Sustainability is the ability of the logistics system to provide long-term supply to troops once the wartime scenario has been assessed.

Sustainability planning must account for the dynamic and probabilistic conditions of war. The unpredictable characteristics of military conflicts can manifest themselves in various ways: changes in intensity or duration of battle, (enemy) units changing locations rapidly at the front lines, consideration of rations due to their vulnerability, damage to resources in the theater, and changes in demand levels due to troop casualties. Characteristically, the lower echelons will be more at risk and more mobile than the upper echelons, which are relatively stable and

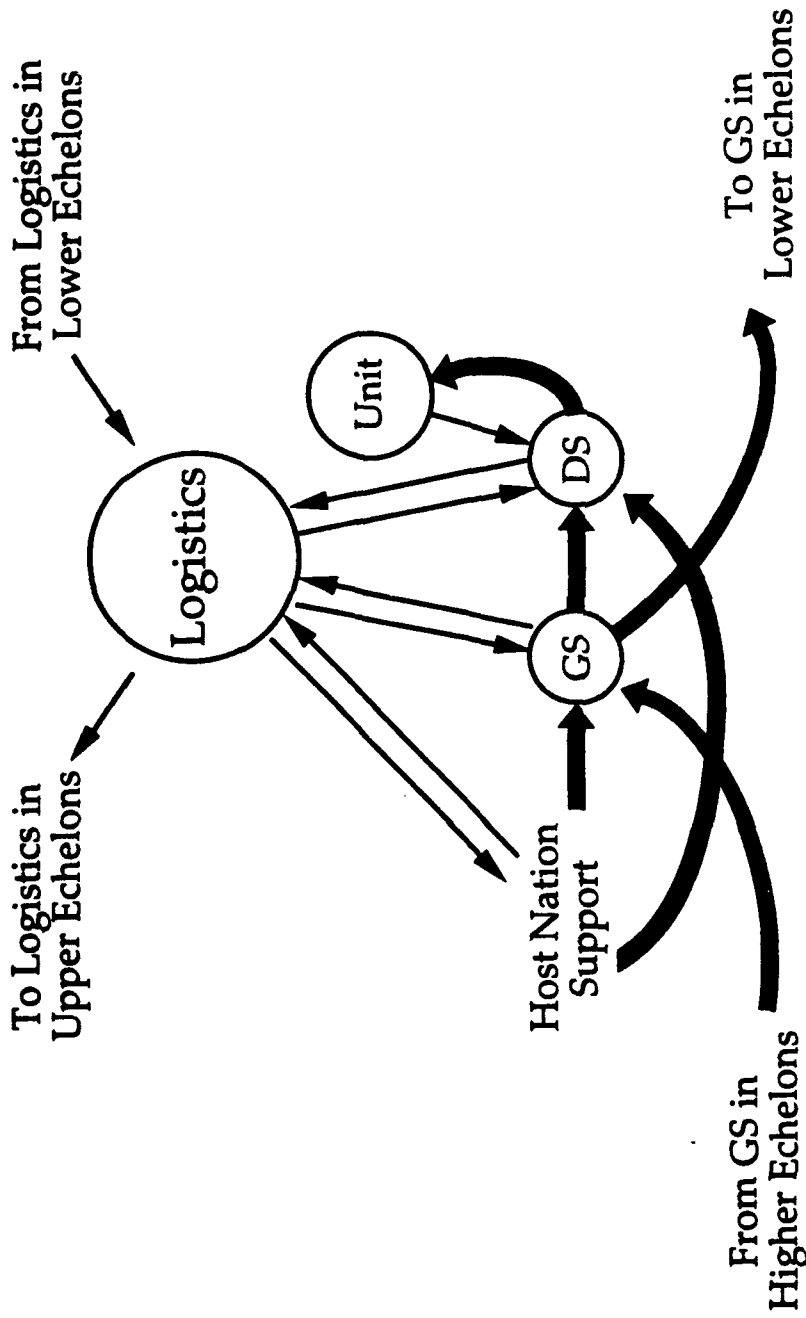


Figure 2.

Generic Structure of a Distribution Node in a Storage Facility Activity Network

GS = General Support      DS = Direct Support

secure. Sustainability is enhanced when the ability to forecast scenarios effectively and anticipate troop movements is increased.

The human element can undermine sustainability due to the pursuit of informal activities, such as trading supplies, or simply by committing (human) errors.

Another factor that can affect ration sustainability adversely is the fact that wartime priorities tend to favor ammunition and fuel over ration supply logistics.

### Model Application Strategy

There are many different methods that are useful for modelling multiechelon distribution problems that have transportation lead times and inventory levels as aspects to consider. It is important to note that although there are many kinds of analytical methods, there is no established method that can cope with every type of multiechelon inventory-distribution problem.

One analytical technique that has gained much popularity over the past decade is just-in-time (JIT) inventory systems analysis. The method of JIT is inapplicable to the ration supply/distribution system for a variety of reasons. First of all, JIT inventory systems are strictly "pull" systems; that is, they respond to demand and they assume that lead time and lead time variability are of an insignificant magnitude compared to how quickly the demands can change. For the ration supply/distribution system, that assumption does not apply since lead times from upper echelons to lower echelons can take days; and demands at the front lines can change much more rapidly. Second, JIT assumes that the cost of set-up, ordering, and transporting are insignificant compared to the holding cost in inventory; again, that assumption clearly does not apply to the ration supply/distribution system. Even though JIT analysis has been undergoing some mathematical research, it is not yet developed enough to be applied to the logistics problem described in this paper.

The modelling technique that was chosen to represent the theater logistics is a minimum-cost flow network model with upper and lower flow bound representation for each arc. The algorithm that is used as part of a heuristic to reduce the operating costs is known as the out-of-kilter algorithm. Using this model, inventory distribution facilities are represented by nodes and transportation routes are represented by arcs in the network. To take the time element of the logistics system into account, the method of dynamic flow was applied to expand the model. The details of the logistics network model are described in the next section.

An overview of the modelling application is shown in Figure 3. The schematic illustrates a cyclical process in which information about the theater (during the military conflict) is fed to the database in the form of network parameter values that define the current situation. The information from the database is fed to the logistics network heuristic for analysis. Orders and instructions that are based on the results of the heuristic analysis are sent to the appropriate parties in the theater from the MMC locations in the echelons. The consequences of the theater activities are sent back to the database and the cycle continues.

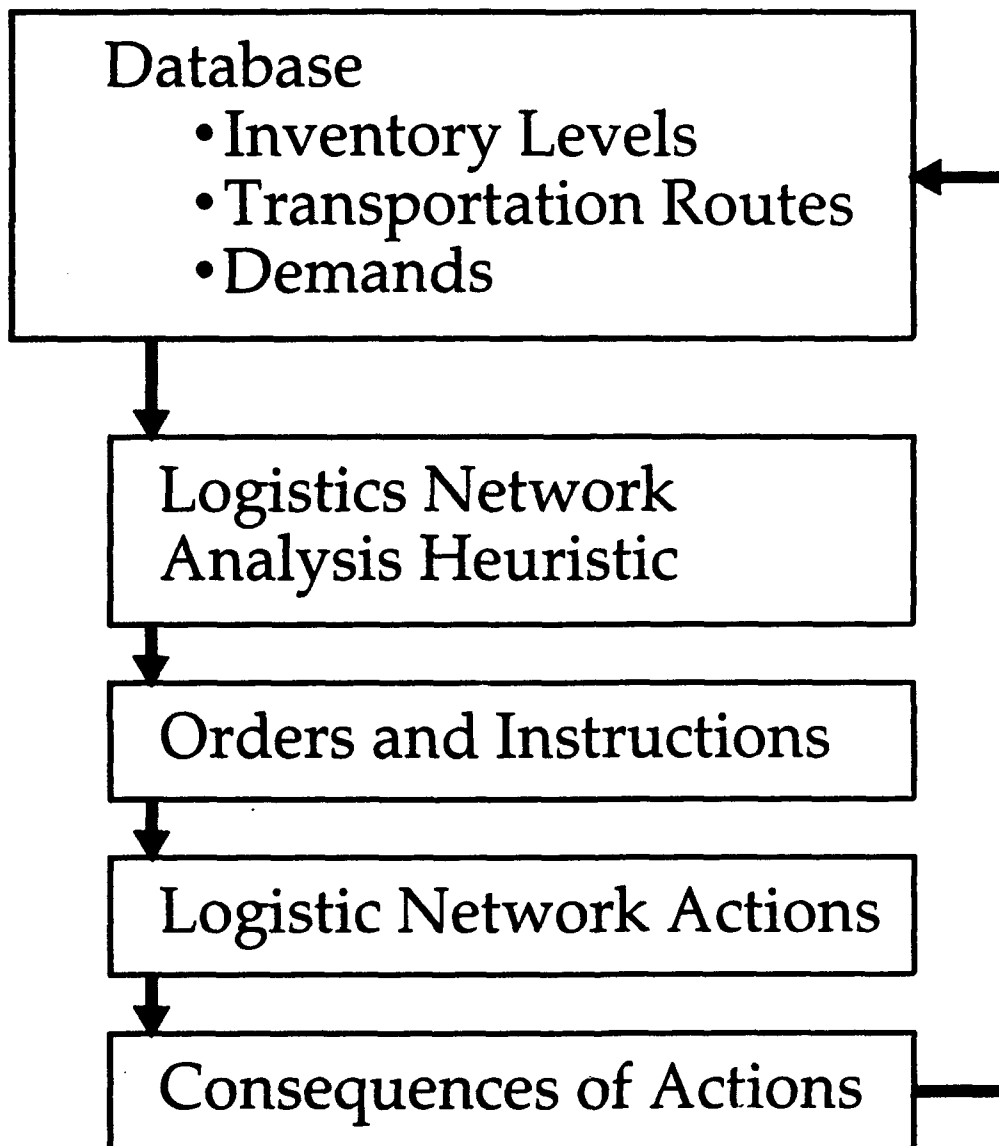


Figure 3.  
Modelling Application Schematic

Note that the cycle that is mentioned above occurs in what will be referred to as a time period. A time period can be considered to equal one day; however, the need for more rapid responses to the changes taking place (particularly in the lower echelons) may warrant the time period and feedback cycle to be reduced to 12 hours.

## LOGISTICS NETWORK MODEL

The multiechelon logistics model is depicted mathematically as a network whose basic components consist of nodes and arcs. The nodes represent either the supply location points, such as Defense Logistics Agency (DLA) depots and/or troop issue subsistence activities (TISAs), or demand points, which are often referred to as ration breakdown locations. The arcs are used to represent the supply routes that exist between the locations (i.e., nodes) in the theatre of operations. The activity that occurs along the arc that connects node  $i$  to node  $j$ , that is, arc  $(i,j)$  is referred to as the flow from  $i$  to  $j$ . The idea is to perform a multiobjective optimization of the network under a constrained set of multicommodity flows. Each commodity,  $k$ , corresponds to a sub-network that is to be optimized by application of a minimum-cost-flow algorithm.

### Purpose

The primary concern of the model is to ensure that the proper amount of each commodity arrives at its appointed destination no later than it is demanded, whenever possible. The supply and demands are represented mathematically by constraints within the network model.

Secondary concerns are represented by the multiobjective function that exists inherently within the cost structure of the model. Reducing various kinds of product flows along the arcs in each of the subnetworks enables a given set of criteria to be emphasized, which is accomplished by manipulating the cost structure (of the arcs) in the network.

The four criteria of fundamental importance within the context of the supply/distribution system are as follows:

(1) Transportation: Reduction of the strain placed on the transportation resources (pertaining to truck and rail oriented supply mechanisms) is important. Since the vehicular resources in the logistics system are typically the most critical factor in the operation of a theater, this criterion should be given the strongest, or nearly the strongest, priority in comparison to the other criteria. Placing a relatively high set of (utility function) costs on the arcs that represent transportation flows in the network will place emphasis on suppressing these flows in comparison with the other types of arc flows in the network.

(2) Inventory Levels: Reduction of storage in pre-position and theater inventories is often useful. Reduction in overall storage levels in theater will tend to speed up rotation of inventory items. An increase in rotation rates is particularly important when perishable items, such as rations, are involved.

(3) Demand: Meeting demands, in terms of preparedness and sustainability can be accomplished by effectively distributing the supply levels throughout the logistics system. Redistribution, during each time period, in response to current and projected demands is useful in avoiding stockouts. The possibility of stockouts is a particularly critical concern at or near the front line of troops (FLOT).



(4) Supply Production Rates and Lead Times: Taking supply source production rates and lead times into consideration is very important. Since each commodity has a limited rate of production (in CONUS plus OCONUS), demand surges can overload the supply sources with production demands. If the upper bounds of supply production rates and the lead times to delivery from each production source are considered, then demands placed directly on production sources can be smoothed out over time.

### Model Structure

There are two major aspects to the construct of the logistics network model. First, the model is dynamic over a countable number of time periods; therefore, an expanded version of the network representation for one time period is used to model the system over the entire planning horizon. Second, the system to be solved is decomposed into what will be referred to as commodity and boundary networks. The volume-constrained boundary network takes into account the cumulative effects of the volume-flow from all the commodity networks. These two aspects, referred to above, will now be described in detail.

### Dynamic Flows Expansion

The concept of the dynamic flow version of the (steady state) logistics network model is more easily grasped when a logistics model having only one commodity is considered. Suppose a situation exists close to the FLOT in which a small section of the logistics system is depicted by Figure 4. Nodes numbered 1 through 4 represent storage facilities. Node S represents an exogenous source; that is, a supply source that might not even exist in the theater itself. The source could be in CONUS or somewhere in OCONUS near the theater. Note that S is directly supplying location 1, in Figure 4 as shown by the arc that connects S and 1. Node S does not have to represent any specific location; the idea is that location 1, in Figure 4, is being supplied by a source or group of source locations that are represented by node S; arc (S,1) conveys this concept. Nodes P and D denote pre-position and demand activities respectively. Neither P nor D represent any location at all; instead, the arcs that emanate from node P and whatever arcs that enter node D represent supply and demand flow activities, respectively. The activities of all the arcs are constrained by upper and lower bounds and regulated by costs; this will be described later in this chapter. The numbers on each arc of Figure 4 denote the number of time periods required for a supply shipment to travel along the arc from node to node. Some of the arcs are assigned (time) values of zero because the activity that the arc represents takes no significant time to occur. For instance, arc (4,D) requires no significant time since it does not represent an actual route in the logistics system. Arc (4,D) exists as an activity to ensure that the total demand of the commodity flowing into node (location) 4 from nodes 2 and 3 does not fall beneath a prescribed lower limit (that is given by the lower bound on arc (4,D)).

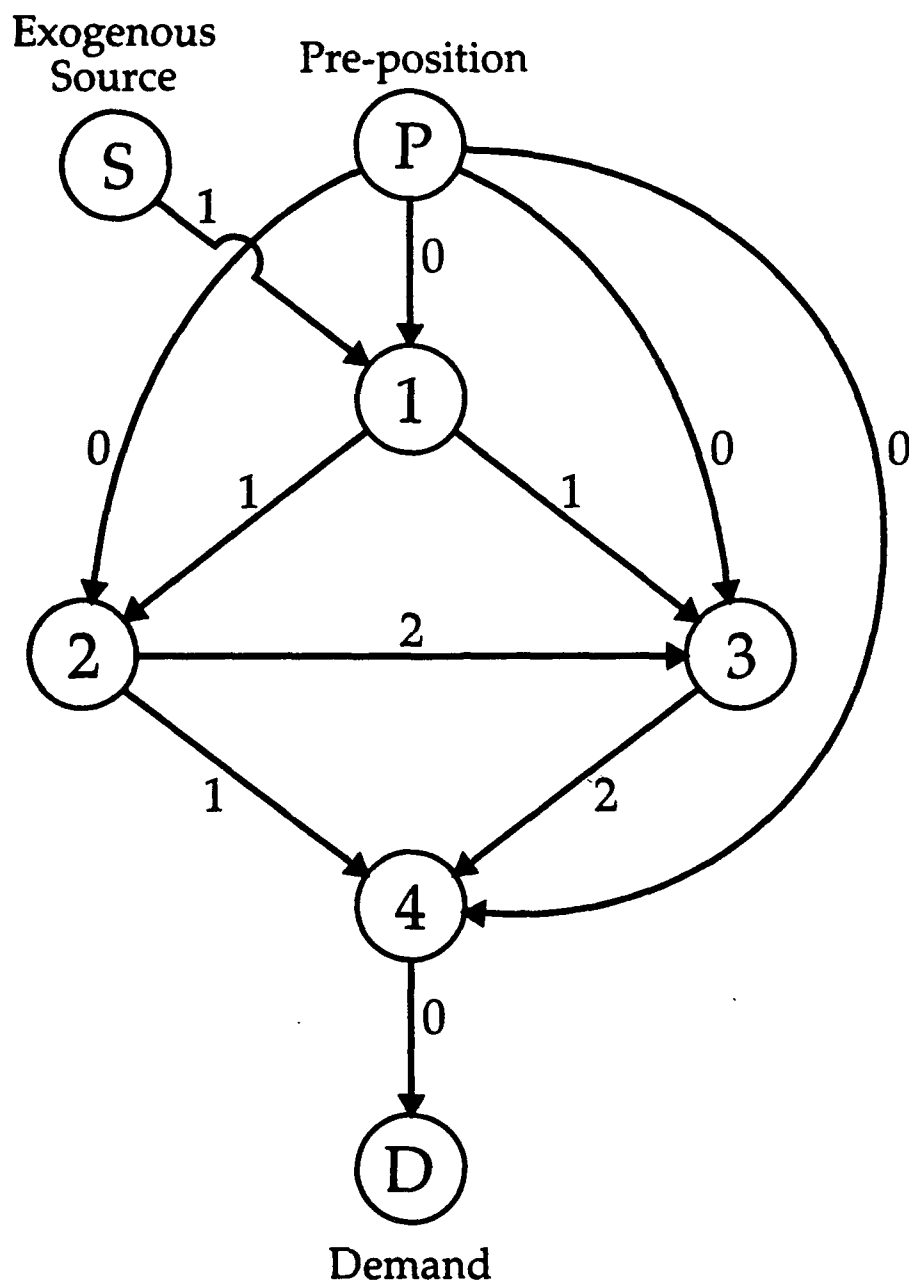


Figure 4.  
 Network Representation of Four Facilities Near the  
 Front Line of Troops with their Supply Routes  
 and Activities  
 Nodes 1 through 4 represent storage facilities.

The dynamic flow version of the graph in Figure 4 is shown in Figure 5. Note that there are nodes that have two numbers in their designations; the first number denotes the time period and the second number corresponds to the identifying number of the node in the graph in Figure 4. The P, S, and D nodes denote the same meanings as in Figure 4. The node referred to in Figure 5 as SS is the supersource node; although SS does not represent any real location the arcs emanating from it have a functional meaning. The only reason node SS is important is due to the fact that network algorithms typically solve graphical representation of networks having one source node, namely SS, and one terminal node, namely D. Networks having multiple sources and multiple terminals (or sinks) are typically modelled using supersource and supersink nodes in the above manner.

The concept of the expanded graph (see Figure 5) can be grasped by considering some brief examples. For instance, consider nodes 1 and 2 in Figure 4. Since one time period is required to send a shipment over arc (1,2), this activity is represented in Figure 5 by arcs from node (1,1) to node (2,2), from (2,1) to (3,2), ..., from node (t,1) to node (t+1,2), etc., from node (5,1) to node (6,2). In another example, consider nodes 2 and 3 in Figure 4. Since two time periods are required to send the commodity along arc (2,3), the representation on the expanded graph are arc from nodes (t,2) to nodes (t+2,3) for  $t = 1, 2, 3$ , and 4 in Figure 5. In general, if arc (A,B) from node A to node B requires  $h$  time periods for a commodity to traverse it, then the representation would be an arc from node (t,A) to node (t+h,B). Over a planning horizon of  $T_H$ , that is, time periods  $1, 2, \dots, T_H$ ,  $t$  would be defined as an integer such that  $1 \leq t \leq T_H - h$ . In Figure 5,  $d_i$  (for  $i = 1, \dots, 6$ ) is the demand for the commodity at time period  $i$ .

The logistics network model takes into consideration five types of activities; they are described as follows (using the notation of the dynamic flows graph):

- (1) Transportation: Arc ((t,A), (t+h,B)) as described above.
- (2) Storage: Arc ((t,A), (t+1,A)) for each time period for  $t = 1, \dots, T_H - 1$ , for location (node) A.
- (3) Demand: Arc ((t,A), D) for demand at node A for  $t = 1, \dots, T_H$ .
- (4) Sources: Arc (S, (t+h,A)) for shipments from the exogenous source to node A for  $t = 1, \dots, T_H - h$ .
- (5) Pre-positioning: Arc [P, (1,A)] for all locations, node A, where pre-positioning is considered.

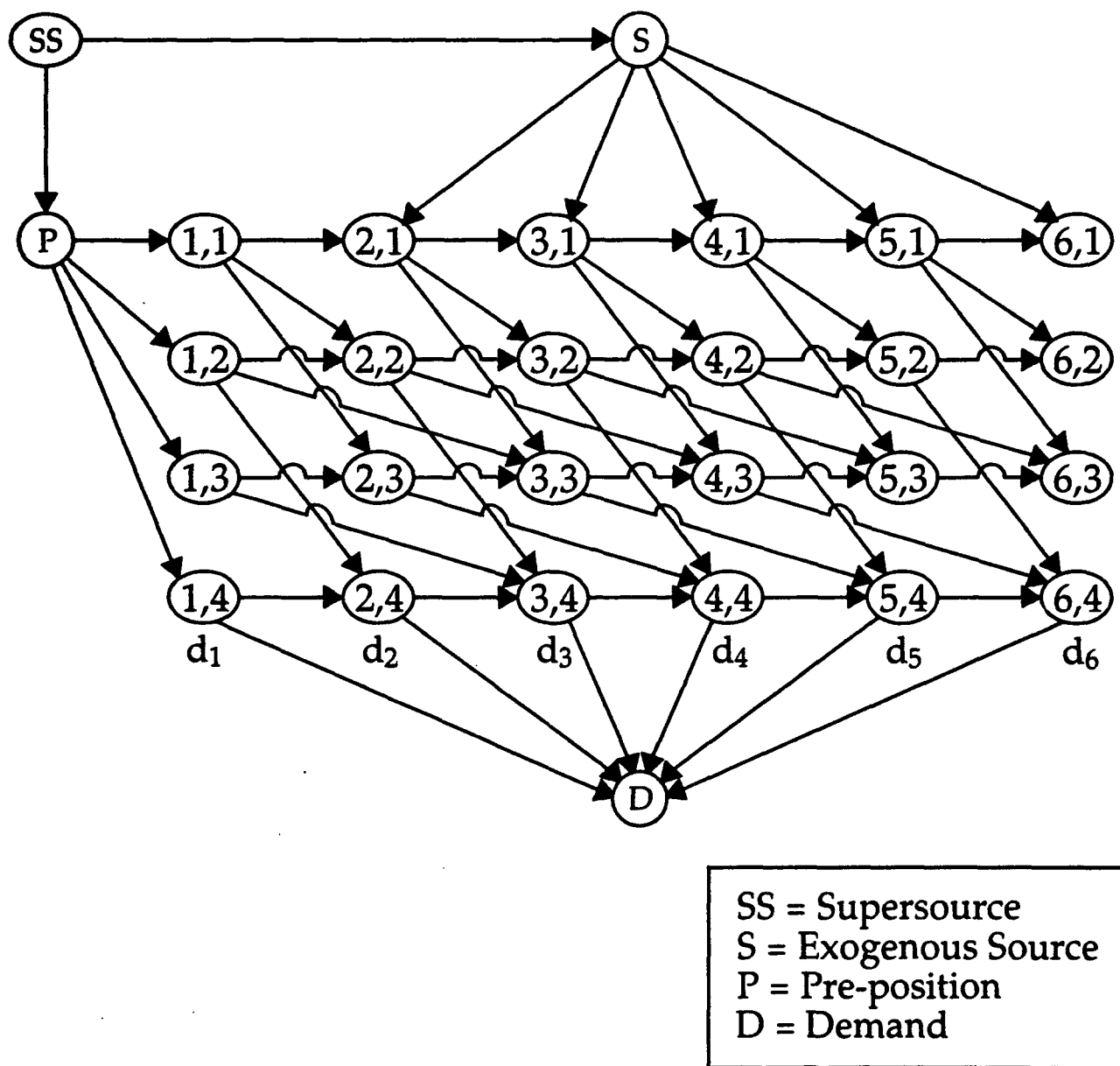


Figure 5.  
 Dynamic Flows Expansion of the Network  
 in Figure 4 for Six Time Periods

In the intermediate nodes, the first indice represents the time period and the second number represents the identifying number of the node from Figure 4.

So far, the arcs involved have been described as routes that require  $h$  time periods to traverse. There are additional parameters that define the arcs that exist mathematically in the model. The parameters of arc  $(i,j)$  for the (expanded) dynamic flows graph are as follows, for each commodity  $k$ :

- (1) The lower bound of the flow is  $l(i,j,k)$ .
- (2) The upper bound of the flow is  $u(i,j,k)$ .
- (3) The cost per unit of the flow is  $c(i,j,k)$ .

The quantity  $f(i,j,k)$  is the number of items (e.g., rations) of commodity  $k$  that flow along arc  $(i,j)$  during the time periods that are represented by arc  $(i,j)$ . It follows that the flow must satisfy the condition:

$$l(i,j,k) \leq f(i,j,k) \leq u(i,j,k) \quad (1)$$

for all arcs  $(i,j)$  for every commodity  $k$ . The upper bound (e.g.,  $u(i,j,k)$ ) is usually referred to as the "capacity" of the arc.

The cost of the flow of arc  $(i,j,k)$  is equal to  $c(i,j,k) f(i,j,k)$ . The objective is to minimize the sum of all the arcs for each commodity  $k = 1, \dots, N_c$ .

$$\text{Minimize } \sum_i \sum_j c(i,j,k) f(i,j,k) , \quad \text{for all } k. \quad (2)$$

The parameters that are assigned to each arc, in the dynamic flows model, distinguish the activity of each of the arcs; the manner in which this is done is described in the following five activities:

(1) For transportation activities, the lower and upper bounds are zero and infinity respectively. The actual number of each commodity,  $k$ , being transported, in itself, is not a limiting factor; the volume (or cube) is a problematic aspect in the logistics system. There are unit costs that can be assigned to each commodity to discourage the use of transportation resources. The unit cost  $c(i,j,k)$  can be based on the mass and volume of commodity  $k$  as well as on considerations that relate to the particulars of the route of arc  $(i,j)$  and any utility functions that the users of the model deem fit to construct.

(2) For storage activities, the lower bound is set to zero, and the upper bound  $u(i,j,k)$  is typically set to infinity, unless other considerations that are inherent to the particular kind of commodity are relevant. As in the transportation activity, it is the unit volume of each commodity that relates to the number of items that can be stored at location  $i$  from one time period  $t$  to the next  $t+1$ . Unit costs can be assigned to the arcs to influence the amount of items being stored. These costs can be related to a utility function that is selected by the users of the model.

(3) For demand activities, the arcs have a finite positive lower bound, recall  $d_i$  in Figure 5 for the beginning of time period  $i$ . Setting the parameter  $l(i,j,k)$  equal to the demand for commodity  $k$  at node  $i$ , where  $j$  corresponds to node  $D$  in Figure 5, forces that demand to be met. The capacity  $u(i,j,k)$  is infinite. The cost is set equal to zero. The demand is what drives the flow through the system. Note that in the marketplace, where these commodities are sold, negative costs would be assigned to encourage sales.

(4) For activities that represent sources, a lower bound of zero and a finite positive upper bound is assigned, that is,  $u(i,j,k) > 0$ . The upper bound is the capacity of the source to produce flow,  $f(i,j,k)$ , where  $i$  corresponds to the source node,  $S$ ,  $j$  corresponds to the receiving facility (node), and  $k$  corresponds to the commodity. The unit cost,  $c(i,j,k)$ , can be set to relate to the market prices. Note that in a major wartime emergency, the costs may be assigned zero, since the lives of the soldiers and the security of regions that are affected most directly could make capitalistic issues insignificant in comparison.

(5) For pre-positioning activities, the arcs would be assigned bound parameter values that are analogous to the values of the source activity arcs, that is, a lower bound of zero and an upper bound  $u(i,j,k) > 0$ . Node  $i$  corresponds to node  $P$  and node  $j$  corresponds to the facility having pre-positioned  $u(i,j,k)$  units of commodity  $k$  just prior to the beginning of the first time period. No costs would be assigned the activity; that is,  $c(i,j,k) = 0$ , unless the effort to remove the items from a pre-positioning mode of storage to an orientation that corresponds to storage that has a higher level of readiness is considered costly for some particular reason.

It is important to note that arcs  $(SS,P)$  and  $(SS,S)$  in Figure 5 have lower and upper bounds of zero and infinity, respectively. Also note that the unit cost of both the arcs is zero.

Note that there could be five types of costs that are associated with the logistics network model; each type of cost corresponds to its associated activity. Because of the different types of costs in the network model, this model can be classified as a multiobjective optimization model in which each objective corresponds to one of the five types of activity costs. As explained in the introduction, the predominant costs in the (military) theater would be transportation and (inventory) storage costs. Pre-positioning costs may be a significant factor in some theater scenarios.

#### Cumulative Effects of Commodity Volume-Flows

A major aspect to the structure of the model is the cumulative influence that the volume-flow of all commodities has with respect to the volumetric capacities of each of the routes in the logistics network. In essence, the issue is cubage. Each commodity has a corresponding volume (or cubage  $V_C(k)$ , for commodities  $k = 1, \dots, N_C$ , where  $N_C$  is the number of different commodities in the logistics system. The cumulative volume-flow through each arc  $(i,j)$  in the dynamic flows network, when summed over all the commodities, is given by equation (3); such a network of cumulative volume-flows is referred to as a boundary (flow) network.

$$v(i,j) = \sum_{k=1}^{N_C} f(i,j,k) V_C(k) \quad (3)$$

Recall that each commodity,  $k$ , has its own corresponding dynamic flows network. When each of the flows  $f(i,j,k)$  in all  $N_C$  dynamic flows networks is determined, the cumulative volume-flows for all arcs  $(i,j)$  in the dynamic flows (boundary) network are computed and compared to their corresponding volumetric flow capacities,  $U_v(i,j)$ , in the dynamic flows (boundary) network for (cumulative) volume-flows.

It is important to note that the dynamic flows network for volumetric flows has exactly the same graphical structure as each of the graphs for the dynamic flows of the commodities.

It is necessary to ensure that none of the volumetric flows,  $v(i,j)$ , exceeds the upper bound,  $U_v(i,j)$ , in the volumetric dynamic flows network. The heuristic for multicommodity flows, which is described in a later section, identifies any violation, such that  $v(i,j) > U_v(i,j)$ , and eliminates it during the next iteration in the heuristic.

Three of the five types of activities in the logistics network have finite volumetric flow capacities; they are transportation, storage, and pre-positioning. Since demand and source activities do not have physical routes that are constrained by any practical considerations with respect to volume (i.e., cubage), the arcs that correspond to these activities are considered to be effectively boundless. Thus,  $U_v(i,j)$  is set to infinity (or an extremely large positive quantity that could not possibly be reached by the volume-flows).

#### Volumetric Flow Limitations

The network flow activities that have finite volumetric capacities have their corresponding upper bounds for various reasons and with differing characteristics. For instance, storage and pre-positioning limits are merely static in comparison to transportation since the number of vehicles at a facility (location) can change very rapidly from one day to the next. Storage and pre-positioning (volumetric) capacities have essentially the same characteristics as each other; they most strongly relate to the internal dimensions (i.e., actual size) of the storage facility that determines their values. For that reason, the volumetric capacities associated with storage facilities are nearly constant, but not always constant since catastrophic events on the battlefield can suddenly reduce these capacities to zero or near zero. If expected capacities are used when it is decided that the risk (or probability) of destruction is significant, then these values will indeed tend to fluctuate more actively from one time period to the next.

Transportation (volumetric) capacities are a much more elaborate issue than those of storage and pre-positioning. Consider a route from location  $i$  to destination  $j$ , within the context shown in Figure 4. The time periods required for the vehicles to get from  $i$  to  $j$  and return to  $i$  are denoted  $T_{i,j}^{(D)}$  and  $T_{i,j}^{(R)}$ , respectively. The superscripts  $D$  and  $R$  stand for destination and return, respectively.  $T_{i,j}^{(D)}$  is the quantity that affects the time dynamic in the dynamic flows network. Note that  $h$ , which was defined in a previous section as the number of time periods required to send a commodity along arc  $(i,j)$ , has the same meaning  $T_{i,j}^{(D)}$ ; their contexts obviously differ from one another. Since the vehicles (usually trucks) are not, as a rule, lent out to other depots, they return  $(T_{i,j}^{(D)} + T_{i,j}^{(R)})$  time periods after leaving location  $i$ . Denote  $V_{i,t}^{(C)}$  as the transportable volume capacity at location  $i$ , and  $V_{i,j,t}^{(R)}$  as the volume that is actually transported from  $i$  to  $j$ ; each starting at the beginning of time period  $t$ . Note that  $V_{i,j,t}^{(R)}$  is not less than the actual volume of products being shipped from facility (location)  $i$  to  $j$  during  $t$ . Thus, location  $i$  will lose  $V_{i,j,t}^{(R)}$  and its volumetric (transport) capacity at time period  $t+1$  would be:

$$V_{i,t+1}^{(C)} = V_{i,t}^{(C)} - V_{i,j,t}^{(R)} \quad (4)$$

if the volumetric gain from the returning vehicles from location  $j$  were not considered. The volumetric gain from the returning vehicles is denoted by  $V_{i,j,q(i,j)}^{(R)}$ . Subscript  $q(i,j)$  is defined as

$$q(i,j) = (t+1) - (T_{i,j}^{(D)} + T_{i,j}^{(R)}) \quad (5)$$

Note that  $q(i,j)$  must be a positive integer. Therefore, taking the returning vehicles into account, the volumetric capacity is;

$$V_{i,t+1}^{(C)} = V_{i,t}^{(C)} - V_{i,j,t}^{(R)} + V_{i,j,q(i,j)}^{(R)} \quad (6)$$



Location  $i$  at time  $t$  and successive time periods,  $t+1, \dots, T_H$  are related to the structure of the dynamic flows network as described in a previous section (of this paper) that discusses the structure of the networks. Therefore, the calculated values of  $V_{i,t}^{(C)}$ , for all  $i$  and  $t$ , can be assigned to the corresponding capacities  $U_v(i,j)$  of the dynamic flows network.

As previously noted, whenever the volumetric flow,  $v(i,j)$ , of the dynamic flows network, exceeds  $U_v(i,j)$ , corrective measures are implemented iteratively by using the heuristic for multicommodity flows. The heuristic is described in detail in the next main section.

### Solution Scheme

Once the commodity flow configuration,  $f(i,j,k)$ , for all arcs  $(i,j)$  and commodities  $k = 1, \dots, N_C$  in the dynamic flows network model are determined, recommendations are made in terms of orders and instructions; refer to Figure 3. The recommendations are based on the solution and can be qualitative as well as quantitative. The recommendations are received by theater logistics personnel and are implemented, either fully or partially, according to the judgement of these personnel.

When the next time period arrives, the current status of the logistics network is fed back to the logistics database and the current status of the distribution supply system is used to update the parameter values of the dynamic flows network model. Again, the flow configurations,  $f(i,j,k)$ , are computed; and the cycle continues as long as needed, as depicted in Figure 3.

## MULTICOMMODITY NETWORK HEURISTIC

The heuristic, which is outlined in this report, uses a network optimization technique that is known as the out-of-kilter algorithm<sup>6,7</sup> as part of its solution methodology. The algorithm, which will be referred to as OKA for brevity, is useful in optimizing minimum cost flow network models. Each commodity in a multicommodity network is solved separately in the heuristic. The rest of the methodology in the heuristic involves the elimination of constraint violations when the volumetric flow  $v(i,j)$  exceeds the volumetric flow capacity  $U_v(i,j)$  of arc  $(i,j)$ . If the violations cannot be removed after a predetermined number of attempts (i.e., cycles) or the commodity network themselves are infeasible initially, the heuristic will terminate without a final solution to the commodity network flows,  $f(i,j,k)$ .

### Characteristics and Advantages of the Out-of-Kilter Algorithm

The out-of-kilter algorithm (OKA) is initiated with a starting feasible solution in the (primal) flow variables. The dual variables are then calculated and the constraint violations in the dual constraint set are assessed. The assessment is made using quantities called kilter numbers.<sup>6,7</sup> Each arc  $(i,j)$  in the graph has its corresponding kilter number. When the flow in the network is augmented such that all the kilter numbers are zero, dual feasibility is achieved; and, thus, the optimal (flow) solution is determined. A set of dual variables, whose values are such that each variable corresponds to each node, uniquely determines the optimizing solution (i.e., analogous to fingerprinting or DNA coding in determining each individual's identity). In performing the iterations with OKA it is found, using the (computer) code that was written as part of the project, that when the largest kilter number is selected to be reduced to zero the algorithm converges significantly more quickly than when the minimum kilter number is chosen to be reduced to zero. Note that kilter numbers can never have negative values.

The advantages of using OKA are considerable. Perhaps the most important reason to consider the use of OKA is that lower (flow) bounds can be defined on each arc; with the minimum-cost-flow algorithm<sup>6</sup> the modeller has to settle for zero as the lower bound. Another useful characteristic of OKA is that noninteger costs (i.e.,  $c(i,j)$  from the previous chapter) can be used without jeopardizing convergence to the optimum after a finite number of iterations. Another advantage in using OKA is that when a change in a parameter value (such as a bound or a unit cost) has been made after the solution of the original network has been obtained, the dual variable values that correspond to the nodes in the network can be used as a starting solution for the altered network. By applying the dual variable values to the altered network, a characteristically rapid convergence is achieved to obtain the new optimum flow configuration, that is, provided the changes did not render the altered network infeasible.

## Heuristic Structure

The application of OKA to the solution of a multicommodity network flow problem is outlined in detail in this section. First, it should be noted that although there are other solution techniques that solve some types of multicommodity flow problems, they are limited mainly to the sums of the flows themselves, that is,  $f(i,j,1) + f(i,j,2) + \dots + f(i,j,N_C)$  and that the methodologies tend to be centered on subject areas covered by linear, integer, and large-scale programming methods. The following heuristic entails a much different approach than the mathematical programming methods and converges to a solution relatively quickly since it uses OKA as a basis for its search routine.

The heuristic will be described here; Figure 6 can be used as a guide. The main idea of the heuristic is to invoke a new and possibly temporary constraint to the arc that exhibits the maximum violation of its upper (volumetric) capacity bound; recall  $U_v(i,j)$ . Note that maximum (constraint) violations were tested in the heuristic with a rate of convergence that was significantly faster than in the case of correcting minimum violations. The added constraint was in the form of a change in one or more of the upper bounds in the commodity networks; it must be emphasized that the added constraints were labeled as not being part of the original formulation with respect to the bound parameter values. These adjusted bounds were determined by using a projection method; the projection method is described in detail in the appendix. After the adjusted upper bounds are added to the commodity networks,  $k$ , for  $k = 1, \dots, N_C$ , OKA is implemented again and the cycle continues, if there are no other complications. There can be many complications, however, in a typical problem of this kind.

The most basic of the complications referred to above is the occurrence of too many bound adjustments added to the commodity networks resulting in the flows being choked off from other arcs in the networks. Under these conditions, the demands that are inherent from the lower (flow) bounds of some of the arcs cannot be met; therefore, infeasibility results in one or more of the over-constrained commodity networks. To remedy that situation, all upper bound (constraint) adjustments prior to the most recent adjustment are deleted. The bounds on the remaining adjusted arc are then "fixed"; this means that the lower bound (in each commodity) is set equal to the adjusted upper bound. This will fix the amount of flow through the arc to a constant value. Note that the term "constraint" in Figure 6 and the terms "bound adjustment" are interchangeable in the description of the heuristic.

The deletion of all but the most recent bound adjustment may happen more than once during the heuristic's search; therefore, the adjusted or "fixed" arc that is left undeleted after all the other bound adjustments are erased may be the same (remaining) adjusted arc as earlier. This is called "cycling." The identity of all the fixed arcs is recorded to detect cycling.

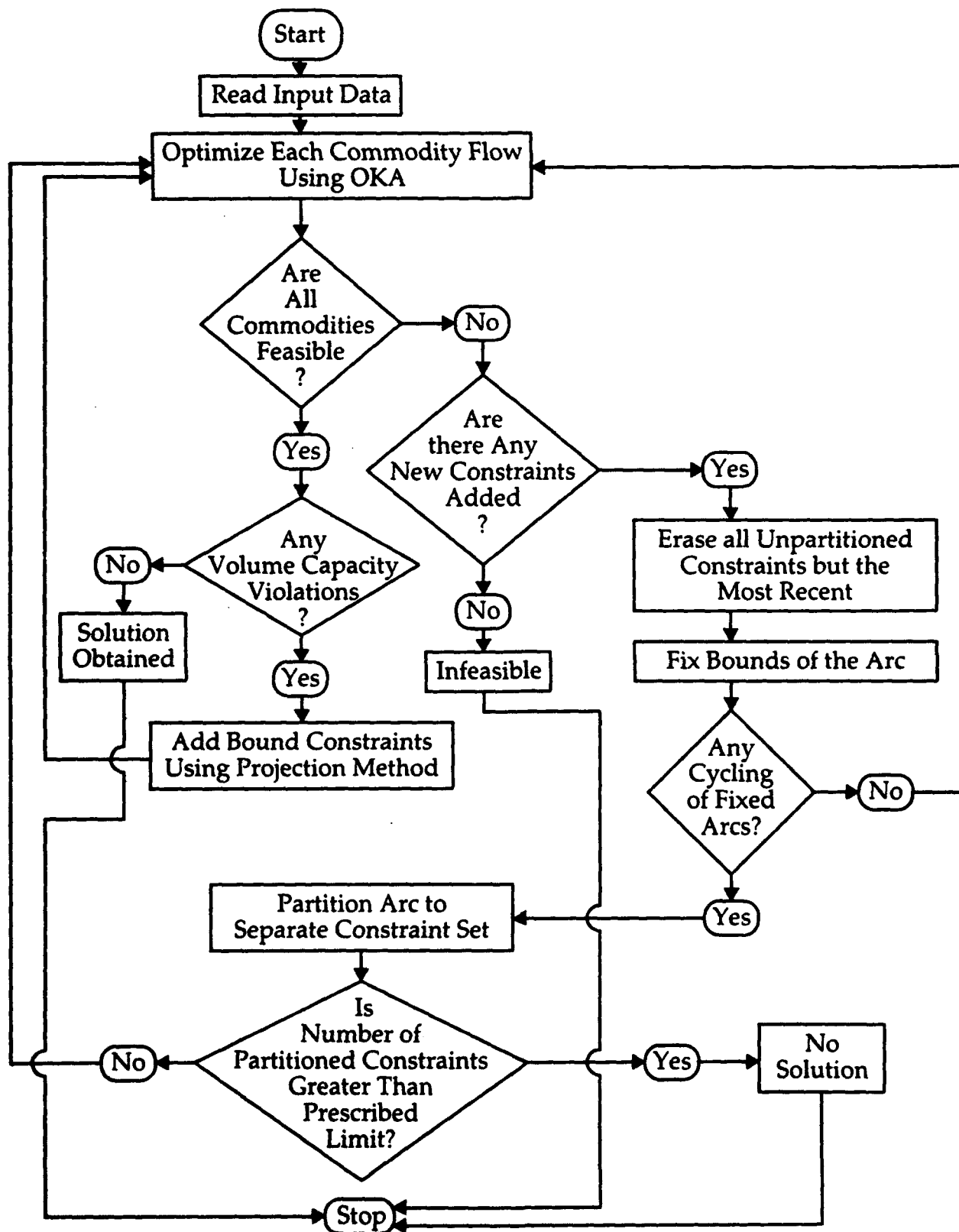


Figure 6.  
Multicommodity Network Heuristic Overview

When an arc is identified as a "cycling arc", its bounds (upper and lower) remain fixed at the same value and the arc is "partitioned" away from its original set of arcs. The partitioning means that the arc joins a collection (i.e., another set) of arcs that will never be altered in any way as long as the heuristic is engaged in solving that particular network problem. If the number of partitioned arcs exceeds a prescribed limit, the heuristic's search is terminated. A recommended value for the upper bound for cycling arcs is three, which is based solely on test runs that were performed. Several experimental network problems that were very tightly constrained yet known to be feasible were performed; all of these networks yielded feasible solutions in two or less discoveries of a cycling arc.

### Solution Characteristics

Since each of the commodity networks is optimized separately and under the presence of upper and lower bound adjustments, the solutions to the multicommodity networks must be considered suboptimal in general. Preliminary trials with some network problems yield results that suggest that the heuristic frequently comes within one percent of the true optimum even though many of the experimental problems were very tightly constrained.

### Data Manipulation for Memory Considerations

Since one of the goals of the project was to allow the software to be used in many different circumstances and at many echelons, an attempt was made to make the software practical for use on a personal computer; for this reason, data files were manipulated by the software in order to save space in storage within the executable code itself.

In spite of the above adjustments, some theater logistics models may have too many facilities for the software to deal with on a personal computer. It may be useful to solve the dynamic flows network model in sections, under the conditions just mentioned. The solution at the boundary to a given section may be used as an input to the boundary of an adjacent section. Lumping facilities that are located relatively close to each other in the theater may be another effective method in circumventing such a problematic issue when a personal computer is employed.

## CONCLUSIONS AND RECOMMENDATIONS

Although a generic model in the form of a network was defined mathematically and a heuristic was developed that is capable of producing a solution to the multicommodity flow network, further experimentation of the model and heuristic may yield additional mathematical insights. Currently, further refinement of the model and heuristic is not a critical issue. The application and extension of the model and heuristic software to either an actual or a small trial problem, which is similar to an actual or a historical theater logistic problem, are the next important steps. Further development would lead toward an application to an existing theater.

Further recommendations can be made to enhance the effectiveness of the model by adding technical extensions. For instance, the development or application of forecasting techniques that use artificial intelligence as a major part of their methodology may provide interesting and useful insights and capabilities. In a similar vein, a set of "learning" parameters that relate to the success/failure history of the logistics model could possibly be developed. The values of the learning parameters could be altered heuristically to influence the future response of the model.

The capability of the model could be extended if input from domain experts is used. Domain experts are individuals who possess a deep understanding of the complexities and intricacies of a particular subject area. Theater MMC personnel and inventory managers may be employed as domain experts for the model; their input may be added to the model in the form of syllogistic rules and guidelines to extend the existing rules base.

If the model and its related software are enhanced by some or all of the ideas that are described above, then it could also have high civilian potential as a useful tool in dealing with inventory and transshipment problems in the private sector.

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This document reports research undertaken at the U.S. Army Natick Research, Development and Engineering Center and has been assigned No. NATICK/TR-14/080 in the series of reports approved for publication.





**APPENDIX**  
**PROJECTION METHOD**

## APPENDIX

### PROJECTION METHOD

The projection method is used to adjust the upper bounds of arcs in the commodity networks so that the volumetric flow  $v(i,j)$  is lowered to equal the volumetric flow capacity  $U_v(i,j)$  of arc  $(i,j)$  in the dynamic flows network.

For convenience in notation, denote vector  $\bar{f}_{i,j}$  as

$$\bar{f}_{i,j} = (f(i,j,1), f(i,j,2), \dots, f(i,j,N_c)) , \quad (A-1)$$

where  $N_c$  is the number of commodities in the network system. Similarly denote  $\bar{V}_c$  as

$$\bar{V}_c = (V_c(1), \dots, V_c(N_c)) . \quad (A-2)$$

Also, denote  $U_v(i,j)$  as  $U_{i,j}$ .

The idea is to project vector  $\bar{f}_{i,j}$  in the direction of negative  $\bar{V}_c$  by a euclidean distance,  $r$ , such that

$$\bar{f}_{i,j}^{(a)} \cdot \bar{V}_c = U_{i,j} . \quad (A-3)$$

This is referred to as (the) projection, and  $\bar{f}_{i,j}^{(a)}$  is referred to as the adjusted vector. Note that the dot product of  $\bar{f}_{i,j}^{(a)}$  and  $\bar{V}_c$  has the same meaning as equation (3) in the section that describes the logistics network model. A complication arises because none of the flow components is allowed to become negative. Because of the non-negativity restriction, the heuristic forces some of the components (i.e., the commodities) of the vector to remain static while other components are allowed to vary as

part of the projection. Define  $\bar{f}_{i,j}^{(v)}$  and  $\bar{f}_{i,j}^{(s)}$  as shown below:

$$\bar{f}_{i,j}^{(v)} = (f(i,j,1), \dots, f(i,j,m), 0, \dots, 0) \quad (A-4)$$

and

$$\bar{f}_{i,j}^{(s)} = (0, \dots, 0, f(i,j,m+1), \dots, f(i,j,N_c)) , \quad (A-5)$$

in which the first  $m$  components are permitted to vary and the remaining terms,  $m+1$  to  $N_c$  are static. Define  $\bar{V}_c^{(v)}$  and  $\bar{V}_c^{(s)}$  in the same (analogous) manner. Thus,

$$\bar{f}_{i,j} = \bar{f}_{i,j}^{(v)} + \bar{f}_{i,j}^{(s)}, \quad (A-6)$$

and

$$\bar{v}_c = \bar{v}_c^{(v)} + \bar{v}_c^{(s)}. \quad (A-7)$$

Considering the concept of the adjusted vector, it follows that  $\bar{f}_{i,j}^{(av)}$  is defined as the adjusted-variable vector such that

$$(\bar{f}_{i,j}^{(av)} \cdot \bar{v}_c^{(v)}) + (\bar{f}_{i,j}^{(s)} \cdot \bar{v}_c^{(s)}) = U_{i,j} \quad (A-8)$$

where

$$\bar{f}_{i,j}^{(a)} = \bar{f}_{i,j}^{(av)} + \bar{f}_{i,j}^{(s)}. \quad (A-9)$$

Vector  $\bar{f}_{i,j}^{(av)}$  is related to  $\bar{f}_{i,j}^{(v)}$  by an adjustment in the negative direction of  $\bar{v}_c^{(v)}$  with distance  $r$  as shown by

$$\bar{f}_{i,j}^{(av)} = \bar{f}_{i,j}^{(v)} - \bar{v}_c^{(v)} r. \quad (A-10)$$

Combining relation (A-8) algebraically with relation (A-10) yields

$$(\bar{f}_{i,j}^{(v)} - \bar{v}_c^{(v)} r) \cdot \bar{v}_c^{(v)} = U_{i,j} - (\bar{f}_{i,j}^{(s)} \cdot \bar{v}_c^{(s)}) \quad (A-11)$$

To solve for  $r$ , do the following manipulations with equation (A-11);

$$(\bar{f}_{i,j}^{(v)} \cdot \bar{v}_c^{(v)}) - (\bar{v}_c^{(v)} \cdot \bar{v}_c^{(v)})r = U_{i,j} - (\bar{f}_{i,j}^{(s)} \cdot \bar{v}_c^{(s)})$$

$$(\bar{v}_c^{(v)} \cdot \bar{v}_c^{(v)})r = (\bar{f}_{i,j}^{(v)} \cdot \bar{v}_c^{(v)}) + (\bar{f}_{i,j}^{(s)} \cdot \bar{v}_c^{(s)}) - U_{i,j}$$

$$(\bar{v}_c^{(v)} \cdot \bar{v}_c^{(v)})r = (\bar{f}_{i,j} \cdot \bar{v}_c) - U_{i,j}$$

$$\text{therefore, } r = \frac{(\bar{f}_{i,j} \cdot \bar{v}_c) - U_{i,j}}{(\bar{v}_c^{(v)} \cdot \bar{v}_c^{(v)})} \quad (A-12)$$

By using equation (A-12) to calculate  $r$ , equation (A-10) can be used to determine  $\bar{f}_{i,j}^{(av)}$ . Next, equation (A-9) is used to determine  $\bar{f}_{i,j}^{(a)}$ .

The components in  $\bar{f}_{i,j}^{(a)}$  are the adjusted capacities of the multicommodity arcs  $(i,j,k)$  for  $k = 1, \dots, N_c$ .